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Gluon-meson duality in the Mean Field Approximation

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Abstract. In a gauge-fixed language gluon-meson duality can be described as the Higgs mechanism for "spontaneous symmetry breaking" of color. We present a mean field computation which suggests that this phenomenon is plausible in QCD. One obtains realistic masses of the light mesons and baryons.

A dual description of QCD in the low momentum or strong coupling region should contain the light meson and baryon degrees of freedom as the relevant fields. This is opposed to the quark-gluon description in the high momentum or weak coupling regime. We have proposed recently [1], [2] that for three light flavors such a dual description indeed exists, with the light gluon fields associated to the light vector meson octet and the nine quark fields to the light baryon octet and a singlet. (For some related ideas[1] see [3], [4].) This picture realizes gluon-meson duality and quark-baryon duality in a straightforward way. Already a very simple form of an effective action can account for realistic masses of the light baryons and the light pseudoscalar and vector meson octets. It also leads to realistic interactions of the Goldstone bosons and predicts the couplings of the vector mesons to pions and baryons in the observed range.

The key ingredient of this picture is a condensate of a suitable quark-antiquark operator which is associated to a color octet. Whereas in [1] we have mainly used a gauge-invariant language in terms of nonlinear fields we concentrate in this note on a (linear) gauge-fixed version (preferably in Landau gauge). In this version the $SU(3)$-color group is "spontaneously broken" by the expectation value of the color octet condensate[2]. The relevant quark-antiquark condensate also transforms as an octet under the vectorlike $SU(3)$ flavor group. Its expectation value conserves a diagonal global $SU(3)$ symmetry of combined color and flavor rotations. This is associated with the physical flavor group of the "eightfold way". Due to this "color-flavor locking" [4] the quark fields transform as an octet and a singlet under the physical flavor group. They carry the appropriate integer electric charges and can be associated with baryons[3]. The octet of gluons acquires a mass through the Higgs mechanism and can be identified with the light vector mesons[4]. Chiral symmetry is spontaneously broken by the octet condensate, as well as by the usual singlet condensate. In the absence of quark masses this leads to eight Goldstone bosons.

In [1] we have mainly discussed the consequences of an assumed expectation value of a color octet quark-antiquark bilinear. Here we present a first investigation if it is plausible that such a vacuum expectation value is generated dynamically in QCD. In this first step we want to identify the mechanisms which could lead to dynamical spontaneous color symmetry breaking. We do not yet intend quantitative estimates.

For this purpose we treat the quantum fluctuations of the light baryons or quarks and the light mesons in the mean field approximation. We consider the non-perturbative region of momenta $q^2 < k^2$, with k an appropriate cutoff (typically $k = 850$ MeV). The form of the effective one-particle irreducible multiquark interactions at the scale k is largely dictated by chiral symmetry, color symmetry and the discrete symmetries P and C, as well as by the known form of the axial anomaly. We include effective interactions involving up to eight quarks or antiquarks. The coefficients of the corresponding invariants in the effective action at the scale k are treated, however, as free parameters. We demonstrate that a reasonable choice of these couplings leads indeed to spontaneous color symmetry breaking and realistic masses for all light particles. This is largely due to the fermion fluctuations with mo-

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[1] In contrast to [3], we consider here the ground state of standard QCD, without additional "fundamental" light-colored scalar fields and without additional gauge interactions. We discuss the vacuum and do not deal with the properties of QCD at very high baryon density

[2] We use the language of spontaneous symmetry breaking similar to the electroweak theory, despite the fact that gauge symmetries are never broken in a strict sense

[3] The gauge fixing is not compatible with the phase transformations corresponding to baryon number. In the gauge-invariant language with nonlinear fields it can be verified that baryons carry indeed three times the baryon number of the quarks [2]

[4] We note, however, that the ϕ-meson and the appropriate mixing of vector mesons is not yet contained in the simplest version. See [2] for an extension with addition of the singlet vector meson

menta $0 < q^2 < k^2$. They are cut off by mass terms in case of chiral symmetry breaking, whereas their contribution disfavors a ground state with absence of octet and singlet condensates. A second crucial ingredient are cubic couplings which reflect the chiral $U_A(1)$ anomaly. A future QCD calculation will have to determine the values of the multiquark couplings at the scale k. Only after this second step one may draw a definite conclusion about the realization of spontaneous color symmetry breaking in QCD.

It is convenient to introduce for the color singlet and octet quark-antiquark bilinears the notation

$$\begin{aligned}\tilde{\varphi}^{(1)}_{ab} &= \bar{\psi}_{L\ ib}\ \psi_{R\ ai} \quad , \quad \tilde{\varphi}^{(2)}_{ab} = -\bar{\psi}_{R\ ib}\ \psi_{L\ ai}\\ \tilde{\chi}^{(1)}_{ij,ab} &= \bar{\psi}_{L\ jb}\ \psi_{R\ ai} - \frac{1}{3}\bar{\psi}_{L\ kb}\ \psi_{R\ ak}\ \delta_{ij}\\ \tilde{\chi}^{(2)}_{ij,ab} &= -\bar{\psi}_{R\ jb}\ \psi_{L\ ai} + \frac{1}{3}\bar{\psi}_{R\ kb}\ \psi_{L\ ak}\ \delta_{ij} \qquad (1)\end{aligned}$$

where i, j, k are color indices and a, b refer to flavor. With respect to the chiral flavor group $SU(3)_L \times SU(3)_R$, the bilinears $\tilde{\varphi}^{(1)}$ and $\tilde{\chi}^{(1)}$ transform as $(\bar{3}, 3)$ whereas $\tilde{\varphi}^{(2)}$ and $\tilde{\chi}^{(2)}$ are in the $(3, \bar{3})$ representation. Parity maps $\tilde{\varphi}^{(1)} \leftrightarrow \tilde{\varphi}^{(2)}$, $\tilde{\chi}^{(1)} \leftrightarrow \tilde{\chi}^{(2)}$ whereas under charge conjugation the transformation is $\tilde{\varphi}^{(i)} \leftrightarrow \tilde{\varphi}^{(i)T}$, $\tilde{\chi}^{(i)}_{ij,ab} \leftrightarrow \tilde{\chi}^{(i)}_{ji,ba}$. At the cutoff we consider an effective Lagrangian of the form[5]

$$\mathcal{L}_k = i\bar{\psi}_{ia}\gamma^\mu\partial_\mu\psi_{ai} - \tilde{U}_k(\tilde{\varphi}, \tilde{\chi}) + \mathcal{L}_{MK} + \mathcal{L}_\eta \qquad (2)$$

with

$$\begin{aligned}&\tilde{U}_k(\tilde{\varphi}, \tilde{\chi})\\ &= 2\lambda_\sigma\tilde{\rho} + 2\lambda_\chi\tilde{\rho}_\chi + \tau_\sigma\tilde{\rho}^2 + \tau_\chi\tilde{\rho}^2_\chi + \tau_\gamma\tilde{\rho}\tilde{\rho}_\chi \qquad (3)\\ &+\zeta\left\{det\tilde{\varphi}^{(1)} + det\tilde{\varphi}^{(2)} + E\left(\tilde{\varphi}^{(1)}, \tilde{\chi}^{(1)}\right) + E\left(\tilde{\varphi}^{(2)}, \tilde{\chi}^{(2)}\right)\right\}\end{aligned}$$

Here the multiquark interactions $\tilde{U}_k$ are expressed in terms of the chirally invariant color singlets

$$\tilde{\rho} = \tilde{\varphi}^{(1)}_{ab}\ \tilde{\varphi}^{(2)}_{ba} \quad , \quad \tilde{\rho}_\chi = \tilde{\chi}^{(1)}_{ij,ab}\ \tilde{\chi}^{(2)}_{ji,ba} \qquad (4)$$

and the 't Hooft term for the chiral anomaly [5] (with coefficient ζ) with

$$E(\tilde{\varphi}, \tilde{\chi}) = \frac{1}{6}\epsilon_{a_1a_2a_3}\epsilon_{b_1b_2b_3}\tilde{\varphi}_{a_1b_1}\tilde{\chi}_{ij,a_2b_2}\tilde{\chi}_{ji,a_3b_3} \qquad (5)$$

We assume that the quantum fluctuations with momenta $q^2 > k^2$ have already been integrated out, such that the remaining functional integral has an effective ultraviolet cutoff k.

In QCD perturbation theory the one-particle-irreducible four-quark interactions $\sim \tilde{\rho}, \tilde{\rho}_\chi$ are generated by box diagrams with

$$\lambda_{\sigma,\chi} = \frac{L_{\sigma,\chi}}{32\pi^2}\frac{g^4}{k^2} \quad , \quad L_\sigma = \frac{23}{9}l^4_3 \quad , \quad L_\chi = \frac{13}{24}l^4_3 \qquad (6)$$

[5] Gluons are incorporated by a covariant derivative of the quarks and an appropriate kinetic term involving their field strength

Here k is the infrared cutoff for the loop momenta and the constant l^4_3 is about one, its value depending on the precise implementation of the cutoff. The coefficients τ_l would correspond to eight quark interactions generated by diagrams with four gluons, $\tau_l \sim g^8/(32\pi^2k^8)$. We will use, however, a scale $k \approx 850$ MeV in the non-perturbative domain and treat for the present work the couplings λ_l and τ_l as free parameters. For the 't Hooft interaction we have used an appropriate Fierz transformation, and ζ is again treated as a free parameter. The terms $\mathcal{L}_\eta$ and $\mathcal{L}_{MK}$ contain sources for the quarks and quark-antiquark bilinears

$$\begin{aligned}\mathcal{L}_\eta &= -\bar{\eta}\psi - \eta\bar{\psi} \qquad (7)\\ \mathcal{L}_{MK} &= -M_{ba}\tilde{\varphi}^{(2)}_{ab} - M^\dagger_{ba}\tilde{\varphi}^{(1)}_{ab} - K_{ij,ab}\tilde{\chi}^{(2)}_{ji,ba} - K^*_{ij,ab}\tilde{\chi}^{(1)}_{ij,ab}\end{aligned}$$

The physical situation corresponds to $\eta = \bar{\eta} = 0$, $K_{ij,ab} = 0$, $M_{ab}(x) = diag(m_u, m_d, m_s)$ with m_q the (current) quark masses.

The first step in a mean field discussion of the multiquark action, $S = \int d^4x\mathcal{L}_k$, is partial bosonization. We introduce a factor of unity (up to an irrelevant overall constant) in the functional integral for the partition function

$$\begin{aligned}&Z[M, K, \eta, \bar{\eta}]\\ &= \int D\psi D\bar{\psi}\exp(-S)\\ &= \int D\psi D\bar{\psi} D\sigma D\xi \exp\Bigg\{ - S - \int d^4x\Big[U_k\Big(\sigma^R_{ab}\\ &\quad -\tilde{\varphi}^R_{ab}, \sigma^I_{ab} - \tilde{\varphi}^I_{ab}, \xi^R_{ij,ab} - \tilde{\chi}^R_{ij,ab}, \xi^I_{ij,ab} - \tilde{\chi}^I_{ij,ab}\Big)\\ &\quad +\mathcal{L}_{MK}\Big(\sigma^R_{ab} - \tilde{\varphi}^R_{ab}, \sigma^I_{ab} - \tilde{\varphi}^I_{ab}, \xi^R_{ij,ab} - \tilde{\chi}^R_{ij,ab}, \xi^I_{ij,ab}\\ &\quad -\tilde{\chi}^I_{ij,ab}\Big)\Big]\Bigg\} \qquad (8)\end{aligned}$$

with

$$\begin{aligned}\tilde{\varphi}^R_{ab} &= \frac{1}{2}\left(\tilde{\varphi}^{(1)}_{ab} + \tilde{\varphi}^{(2)}_{ba}\right) \quad ,\\ \tilde{\varphi}^I_{ab} &= -\frac{i}{2}\left(\tilde{\varphi}^{(1)}_{ab} - \tilde{\varphi}^{(2)}_{ba}\right)\\ \tilde{\chi}^R_{ij,ab} &= \frac{1}{2}\left(\tilde{\chi}^{(1)}_{ij,ab} + \tilde{\chi}^{(2)}_{ji,ba}\right) \quad ,\\ \tilde{\chi}^I_{ij,ab} &= -\frac{i}{2}\left(\tilde{\chi}^{(1)}_{ij,ab} - \tilde{\chi}^{(2)}_{ji,ba}\right) \qquad (9)\end{aligned}$$

The function U_k is determined by the requirement that $U_k(-\tilde{\varphi}^R, -\tilde{\varphi}^I, -\tilde{\chi}^R, -\tilde{\chi}^I) = \tilde{U}_k(\tilde{\varphi}^R, \tilde{\varphi}^I, \tilde{\chi}^R, \tilde{\chi}^I)$ is given by (3) expressed in the appropriate variables.

The effective Lagrangian of the bosonized theory

$$\mathcal{L}^{(B)} = \mathcal{L}(\tilde{\varphi}, \tilde{\chi}) + U_k(\sigma - \tilde{\varphi}, \xi - \tilde{\chi}) + \mathcal{L}_{MK}(\sigma - \tilde{\varphi}, \xi - \tilde{\chi}) \quad (10)$$

is next expanded in powers of $\tilde{\varphi}$ and $\tilde{\chi}$. The terms involving no fermion fields result in masses, sources, and interactions for the scalars

$$\begin{aligned}\mathcal{L}_s = U_k(\sigma, \xi) &- Tr(M^\dagger\sigma + M\sigma^\dagger)\\ &-(K^*_{ij,ab}\xi_{ij,ab} + K_{ij,ab}\xi^*_{ij,ab}) \qquad (11)\end{aligned}$$

The potential U_k obtains from (3) by the replacements $\tilde{\varphi}^{(1)}_{ab} \to \sigma_{ab}$, $\tilde{\varphi}^{(2)}_{ab} \to \sigma^\dagger_{ab}$, $\tilde{\chi}^{(1)}_{ij,ab} \to \xi_{ij,ab}$, $\tilde{\chi}^{(2)}_{ij,ab} \to \xi^*_{ji,ba}$, $\tilde{\rho} \to Tr\ \sigma^\dagger\sigma$, $\tilde{\rho}_\chi \to \xi^*_{ij,ab}\xi_{ij,ab}$, where we have combined $\sigma_{ab} = \sigma^R_{ab} + i\sigma^I_{ab}$, $\xi_{ij,ab} = \xi^R_{ij,ab} + i\xi^I_{ij,ab}$. Furthermore, the sign of the term ζ is switched. The chiral and discrete transformation properties carry over from $\tilde{\varphi}, \tilde{\chi}$ to σ, ξ and U_k is therefore invariant under the corresponding symmetries. The inclusion of the eight-quark interactions in (3) guarantees that for positive τ_l the functional integral (8) is well defined and U_k is bounded from below. On the other hand, the terms in the expansion of $U_k + \mathcal{L}_{MK}$ which involve only fermion fields precisely cancel the multiquark interactions $-\tilde{U}_k$ and the source $\mathcal{L}_{MK}$ for the quark bilinears. Chiral symmetry breaking due to quark masses appears now in the form of a linear source term for the scalar fields (11). The terms linear in the quark bilinears give rise to Yukawa-type interactions, involving a quark-antiquark pair and one or several scalar fields. Finally, the terms $\sim \zeta$ and τ_l also produce interactions between four or six fermions and one or two scalars. In our mean field approximation these residual interactions involving more than two quark or antiquark fields are neglected.

We want to perform the remaining fermionic functional integral for a scalar background field which preserves a "diagonal" vector-like $SU(3)$ symmetry and is invariant under C and P

$$\sigma_{ab} = \bar{\sigma}\delta_{ab} \quad , \quad \xi_{ij,ab} = \frac{1}{\sqrt{6}}\bar{\xi}\left(\delta_{ia}\delta_{jb} - \frac{1}{3}\delta_{ij}\delta_{ab}\right) \tag{12}$$

For this configuration[6] the effective Lagrangian (10) is given by

$$\mathcal{L}^{(B)} = U_{cl}\left(\bar{\sigma},\bar{\xi}\right) + i\bar{\psi}_{ia}\gamma^\mu\partial_\mu\psi_{ai} + M_8\left(\bar{\sigma},\bar{\xi}\right)\bar{\psi}^{(8)}_{ia}\bar{\gamma}\psi^{(8)}_{ai} + M_1\left(\bar{\sigma},\bar{\xi}\right)\bar{\psi}^{(1)}\bar{\gamma}\psi^{(1)} \tag{13}$$

with $\bar{\gamma}$ the Euclidean analogue[7] of γ^5. The fermionic part involves a mass term for the $SU(3)$-singlets and octets

$$\psi^{(1)} = \frac{1}{\sqrt{3}}\psi_{aa} \quad , \quad \psi^{(8)}_{ai} = \psi_{ai} - \frac{1}{\sqrt{3}}\psi^{(1)}\delta_{ai} \tag{14}$$

where the latter is identified with the light baryon octet. One finds

$$M_8 = 2\lambda_\sigma\bar{\sigma} - \frac{2}{3\sqrt{6}}\lambda_\chi\bar{\xi} - \zeta\left(\bar{\sigma}^2 - \frac{2}{27}\bar{\xi}^2 + \frac{1}{9\sqrt{6}}\bar{\sigma}\bar{\xi}\right) + 6\tau_\sigma\bar{\sigma}^3 + \frac{4}{3}\tau_\gamma\bar{\xi}^2\bar{\sigma} - \frac{1}{\sqrt{6}}\tau_\gamma\bar{\sigma}^2\bar{\xi} - \frac{8}{9\sqrt{6}}\tau_\chi\bar{\xi}^3 \tag{15}$$

$$M_1 = 2\lambda_\sigma\bar{\sigma} + \frac{16}{3\sqrt{6}}\lambda_\chi\bar{\xi} - \zeta\left(\bar{\sigma}^2 - \frac{2}{27}\bar{\xi}^2 - \frac{8}{9\sqrt{6}}\bar{\sigma}\bar{\xi}\right) + 6\tau_\sigma\bar{\sigma}^3 + \frac{4}{3}\tau_\gamma\bar{\xi}^2\bar{\sigma} + \frac{8}{\sqrt{6}}\tau_\gamma\bar{\sigma}^2\bar{\xi} + \frac{64}{9\sqrt{6}}\tau_\chi\bar{\xi}^3 \tag{16}$$

The classical scalar potential reads

$$U_{cl} = -2m\bar{\sigma} + 6\lambda_\sigma\bar{\sigma}^2 + \frac{8}{3}\lambda_\chi\bar{\xi}^2 - 2\zeta\bar{\sigma}^3 + \frac{4\zeta}{9}\bar{\sigma}\bar{\xi}^2 + 9\tau_\sigma\bar{\sigma}^4 + \frac{16}{9}\tau_\chi\bar{\xi}^4 + 4\tau_\gamma\bar{\sigma}^2\bar{\xi}^2 \tag{17}$$

with $m = m_u + m_d + m_s$. The sign of ζ may be positive or negative[8] and we discuss below acceptable scenarios for both cases. For positive couplings λ_l, τ_l the classical potential typically[9] has its minimum for $\bar{\xi} = 0$. Then spontaneous symmetry breaking of color symmetry can only be induced by the low momentum fluctuations.

The fermionic functional integral is easily evaluated and gives a contribution to the effective scalar potential

$$U(\bar{\sigma},\bar{\xi}) = U_{cl}(\bar{\sigma},\bar{\xi}) + \Delta_q U(\bar{\sigma},\bar{\xi}) \tag{18}$$

For a sharp ultraviolet cutoff k this reads

$$\Delta_q U = -\frac{1}{8\pi^2}\int_0^{k^2} dx x[8\ln(x + M_8^2) + \ln(x + M_1^2)] \tag{19}$$

and one observes that $\Delta_q U$ respects the (accidental?) symmetry[10] of the classical potential $\bar{\xi} \to -\bar{\xi}$. The quark fluctuations tend to destabilize the minimum of U_{cl} at $\bar{\sigma} = \bar{\xi} = 0$ for $m = 0$, since nonzero values of M_8, M_1 are preferred. They give a negative contribution to the quadratic term obtained by an expansion of $\Delta_q U$ for small $\bar{\sigma}$ and $\bar{\xi}$

$$\Delta_q U = -\frac{k^2}{2\pi^2}\left(9\lambda_\sigma^2\bar{\sigma}^2 + \frac{4}{3}\lambda_\chi^2\bar{\xi}^2\right) + \ldots \tag{20}$$

For $\lambda_\sigma k^2 > 4\pi^2/3$, $\lambda_\chi k^2 > 4\pi^2$ they overwhelm the classical mass terms $6\lambda_\sigma\bar{\sigma}^2$, $(8/3)\lambda_\chi\bar{\xi}^2$ for $\bar{\sigma}$ and $\bar{\xi}$, respectively. We conclude that the quark fluctuations are

[6] We take both $\bar{\sigma}$ and $\bar{\xi}$ to be real such that P is conserved. In principle, $\bar{\xi}$ could have a relative phase as compared to $\bar{\sigma}$. A purely imaginary $\bar{\xi}$ is favored by the anomaly term in U_k if $\zeta\bar{\sigma}$ is positive at the minimum. From this point of view one may prefer a negative value of ζ. On the other hand, invariants of the type $\tilde{\rho}\tilde{\rho}_\chi - \frac{1}{4}(\sigma^\dagger_{ab}\xi_{ijbc}\sigma^\dagger_{cd}\xi_{jida} + c.c.)$ can also favor real $\bar{\xi}$. We have not included in the present discussion invariants which vanish identically for the configuration (12) and real $\bar{\sigma}, \bar{\xi}$ as the one above. Parity conservation remains an issue in our approach. In order to settle this issue one needs to include invariants which contribute to a background field (12) with an arbitrary phase for $\bar{\xi}$. Also the quantum fluctuations will have to be treated in this more general setting. At the present stage the positive mass term $M^2_{\eta'}$ for the η'-meson (see below) is a strong hint that the ground state preserves parity

[7] For our conventions see [6]

[8] Only the relative phase between ζ and m is relevant. Conservation of C, P requires that the phases of ζ and m are equal up to a minus sign and we take both ζ and m real. We choose a phase convention such that the expectation value of $\bar{\sigma}$ is positive and concentrate on the case $m_q > 0$

[9] This clearly holds if $\zeta\bar{\sigma}$ is positive at the minimum. For negative $\zeta\bar{\sigma}$ one may also envisage the possibility that already the effective action (2) has its minimum for $\bar{\xi} \neq 0$

[10] This is not trivial since an invariant $\epsilon_{a_1a_2a_3}\ \epsilon_{b_1b_2b_3}\ \epsilon_{i_1i_2i_3}\ \epsilon_{j_1j_2j_3}\ \chi_{i_1j_1,a_1b_1}\ \chi_{i_2j_2,a_2b_2}\ \chi_{i_3j_3,a_3b_3} + c.c.$ is consistent with color and chiral symmetries as well as P and C

the main driving force for spontaneous chiral symmetry breaking. Furthermore, the cubic "anomaly terms" favor the condensates even in presence of small enough positive quadratic terms at the origin. In mean field theory one has to determine the minimum of U. We have done this by numerically solving the field equations. In this context we note the simple closed form of the contribution from $\Delta_q U$, namely

$$\begin{aligned}\frac{\partial}{\partial\bar{\sigma}}\Delta_q U &= 8A_8\frac{\partial M_8}{\partial\bar{\sigma}} + A_1\frac{\partial M_1}{\partial\bar{\sigma}} \quad ,\\ \frac{\partial}{\partial\bar{\xi}}\Delta_q U &= 8A_8\frac{\partial M_8}{\partial\bar{\xi}} + A_1\frac{\partial M_1}{\partial\bar{\xi}}\\ A_{1,8} &= \frac{M_{1,8}^3}{4\pi^2}\left\{\ln\left(1+\frac{k^2}{M_{1,8}^2}\right) - \frac{k^2}{M_{1,8}^2}\right\}\end{aligned} \tag{21}$$

For a large range of couplings we find indeed a nonzero expectation value for the octet condensate $\bar{\xi}$!

As long as the effective couplings λ_l, τ_l, ζ are not computed from QCD, the predictive power of the present approach is limited. We will be satisfied here by presenting a set of perhaps reasonable values for these couplings for which a realistic phenomenology can be obtained. Some of the parameters are tuned to fit with observed masses. We choose $k = 850$ MeV equal to the average of the ρ-meson or gluon mass. The renormalization scale for the quark mass is taken at $\mu = 1$ GeV in the vicinity of the baryon masses, and we take for the quark mass sum $m(\mu) = 220$ MeV. Our selected parameters are

$$\begin{aligned}&\lambda_\sigma k^2 = 53(63,57), \quad \lambda_\chi k^2 = 0.47(0.9,5.8),\\ &\zeta k^5 = 356(32,-73.5), \quad \tau_\sigma k^8 = 4700(185,8060),\\ &\tau_\chi k^8 = 87(44,314), \quad \tau_\gamma k^8 = 5200(350,1570)\end{aligned} \tag{22}$$

Here the numbers in brackets refer to two sets with inclusion of fluctuations of the gauge bosons and the pseudoscalar octet, as discussed below. For these values the minimum of U occurs for

$$\langle\bar{\sigma}\rangle = \frac{1}{2}(235 \text{ MeV})^3 \quad , \quad \langle\bar{\xi}\rangle = \frac{1}{2}(400(470,280) \text{ MeV})^3 \tag{23}$$

and we observe the direct relation of $\langle\bar{\sigma}\rangle$ with the (singlet) quark-antiquark condensate[11] and the parameters B and f of chiral perturbation theory (f is the meson decay constant)

$$\langle\bar{\sigma}\rangle = -\frac{1}{2}\langle\bar{\psi}\psi\rangle(\mu) = \frac{1}{2}B(\mu)f^2 \tag{24}$$

The average mass of the lightest baryon octet and singlet obtains by evaluating (15), (16) for the expectation value (23). For our parameters they coincide with the observed value of the average octet mass and an arbitrary fixed singlet mass[12].

$$M_8 = 1.15 \text{ GeV} \quad , \quad M_1 = 1.4 \text{ GeV} \tag{25}$$

It is instructive to determine the masses of the most important bosonic excitations for our parameter set. For a computation of the scalar masses one needs the scalar wave function renormalizations which have not been computed so far. Altogether, the fields σ and ξ contain 162 real scalars which transform under $SU(3)$, P and C as

$$\begin{aligned}&2\times(1^{-+}+1^{++}+8^{-+}+8^{++}+8^{--}+8^{+-})\\ &\quad +10^{--}+10^{+-}+\bar{10}^{--}+\bar{10}^{+-}+27^{-+}+27^{++}\end{aligned} \tag{26}$$

One octet is absorbed by the Higgs mechanism into the longitudinal component of the massive gluons. The representations $10, \bar{10}, 27$ contain scalars with electric charge two. Many of these states may be too broad to be detected experimentally as resonances. Of particular interest are the lightest pseudoscalar octet 8^{-+} which corresponds to massless Goldstone bosons for $m_q \to 0$, and the lightest pseudoscalar singlet 1^{-+} associated with the η'-meson. They are most easily described in a nonlinear representation [2]

$$\begin{aligned}\sigma_{ab} &= \langle\bar{\sigma}\rangle U_{ab} \quad ,\\ \xi_{ij,ab} &= \frac{1}{\sqrt{6}}\langle\bar{\xi}\rangle\left(U_{ai}^{1/2}U_{jb}^{1/2} - \frac{1}{3}\delta_{ij}U_{ab}\right),\\ U &= \exp\left(-\frac{i}{3}\theta\right)\exp\left(i\frac{\Pi^z\lambda_z}{f}\right) \quad ,\\ U^\dagger U &= 1\end{aligned} \tag{27}$$

If we parametrize the effective scalar kinetic terms[13] by

$$\mathcal{L}_{s,kin} = Z_\sigma\partial^\mu\sigma_{ab}^*\partial_\mu\sigma_{ab} + Z_\xi\partial^\mu\xi_{ij,ab}^*\partial_\mu\xi_{ij,ab} \tag{28}$$

the kinetic term for the pseudoscalar nonet reads

$$\begin{aligned}\mathcal{L}_{U,kin} &= \left(Z_\sigma\langle\bar{\sigma}\rangle^2 + \frac{7}{36}Z_\xi\langle\bar{\xi}\rangle^2\right) \quad Tr\partial^\mu U^\dagger\partial_\mu U\\ &= \frac{1}{4}f^2 \; Tr\partial^\mu U^\dagger\partial_\mu U\end{aligned} \tag{29}$$

An average meson decay constant $f = 106$ MeV corresponds to

$$\begin{aligned}&Z_\sigma + \frac{7}{36}Z_\xi(\bar{\xi}/\bar{\sigma})^2 = Z_\sigma(1+x) = (350 \text{ MeV})^{-4},\\ &x = \frac{7}{36}\frac{\langle\bar{\xi}\rangle^2}{\langle\bar{\sigma}\rangle^2}\frac{Z_\xi}{Z_\sigma}\end{aligned} \tag{30}$$

[11] In our Euclidean conventions $\langle\bar{\psi}\psi\rangle$ corresponds to the expectation value of $\bar{\psi}\bar{\gamma}\psi$

[12] The baryon singlet is presumably very broad and it is not obvious if it should be associated with an observed resonance. The lowest mass resonance with the correct quantum numbers occurs at 1.6 GeV

[13] Presumably the dominant contribution to the kinetic terms is induced by quantum fluctuations with $q^2 < k^2$. It can also be computed in the mean field approximation. Integrating out first the fermions will also induce an effective gauge coupling between the gluons and the color octet ξ, replacing $\partial_\mu\xi$ by a suitable covariant derivative

If this is the case, chiral symmetry guarantees realistic masses for the pions, kaons, and the η-meson. Neglecting $SU(3)$-violating effects for the expectation values they are given by

$$\begin{aligned} M_\pi^2 &= 2\langle\bar\sigma\rangle(m_u+m_d)/f^2, \\ M_K^2 &= 2\langle\bar\sigma\rangle(m_u+m_s)/f^2 \end{aligned} \tag{31}$$

independently of details of the potential [7]. For realistic f and appropriate quark mass ratios $m_u/m_s, m_d/m_s$ the expectation values (23) lead to the observed light pseudoscalar meson masses.

If we associate the η'-meson with θ in (27), neglect its mixing with other mesons with the same quantum numbers and assume that its kinetic term is dominated by (29) we find

$$M_{\eta'}^2 = \frac{36\zeta}{f^2}\left(\bar\sigma^3 - \frac{2}{9}\bar\sigma\bar\xi^2 + \Delta_{\eta'}\right) + m_g^2 \tag{32}$$

where $m_g = 410$ MeV is the contribution from explicit chiral symmetry breaking due to the current quark masses and

$$\begin{aligned} \Delta_{\eta'} = 8A_8 &\left(\bar\sigma^2 - \frac{2}{27}\bar\xi^2 + \frac{1}{9\sqrt6}\bar\sigma\bar\xi\right)\cdot\left(2\lambda_\sigma\bar\sigma - \frac{2}{3\sqrt6}\lambda_\chi\bar\xi\right. \\ &\left.+6\tau_\sigma\bar\sigma^3 + \frac43\tau_\gamma\bar\xi^2\bar\sigma - \frac{1}{\sqrt6}\tau_\gamma\bar\sigma^2\bar\xi - \frac{8}{9\sqrt6}\tau_\chi\bar\xi^3\right) \\ &+A_1\left(\bar\sigma^2 - \frac{2}{27}\bar\xi^2 - \frac{8}{9\sqrt6}\bar\sigma\bar\xi\right)\cdot\left(2\lambda_\sigma\bar\sigma + \frac{16}{3\sqrt6}\lambda_\chi\bar\xi\right. \\ &\left.+6\tau_\sigma\bar\sigma^3 + \frac43\tau_\gamma\bar\xi^2\bar\sigma + \frac{8}{\sqrt6}\tau_\gamma\bar\sigma^2\bar\xi + \frac{64}{9\sqrt6}\bar\xi^3\right) \end{aligned} \tag{33}$$

arises from the θ-dependence of the fermionic fluctuation determinant giving rise to $\Delta_q U$. As is should be, $M_{\eta'}$ vanishes for zero quark masses and vanishing chiral anomaly ($\zeta = 0$). For our parameters (32) yields the observed value $M_{\eta'} = 960$ MeV.

The gluons are coupled to the colored scalar fields ξ and therefore acquire a mass through the Higgs mechanism. We identify this mass with the average mass of the light vector meson octet M_ρ =850 MeV. Denoting the effective gauge coupling by g one obtains

$$M_\rho = g\ Z_\xi^{1/2}\langle\bar\xi\rangle = \frac{3}{\sqrt7}\left(\frac{x}{1+x}\right)^{1/2} gf \tag{34}$$

For our set of parameters one finds a large value of x if Z_σ/Z_ξ is not too large For $Z_\sigma/Z_\xi = 0.44(3, 0.13)$ this yields[14]

$$\begin{aligned} x &= 10.9(4,4), \qquad g = 7.4(7.9,7.9), \\ Z_\xi &= (530(690,320)\ \text{MeV})^{-4} \end{aligned} \tag{35}$$

[14] The relation to the parameters used in [1] is given by $m_\phi^2 = 2\lambda_\sigma/Z_\sigma = (5.1(3.6, 3.56)\ \text{GeV})^2$, $m_\chi^2 = 2\lambda_\chi/Z_\xi = (0.32(0.75, 0.42)\ \text{GeV})^2$, $h = 2\lambda_\sigma Z_\sigma^{-1/2} = 62(48, 46)$, $\tilde h = 2\lambda_\chi Z_\xi^{-1/2} = 0.36(1.2, 1.8)$, $\tilde\epsilon_\phi = m_\phi^2/(h^2k^2) = (2\lambda_\sigma k^2)^{-1} = 0.009(0.008, 0.009)$, $\tilde\epsilon_\chi = m_\chi^2/(\tilde h^2 k^2) = (2\lambda_\chi k^2)^{-1} = 1(0.55, 0.086)$, with $\varphi = Z_\sigma^{1/2}\sigma$, $\chi = Z_\xi^{1/2}\xi$

One may want to include the effects of gauge boson and scalar fluctuations in the effective potential. In the mean field approximation one finds

$$\begin{aligned} \Delta_g U &= \frac{3}{4\pi^2}\int_0^{k^2} dx\ x\ ln\left(x + g^2 Z_\xi\bar\xi^2\right) \\ \Delta_s U &= \frac{1}{32\pi^2}\sum_s\int_0^{k^2} dx\ x\ ln(x + M_s^2) \end{aligned} \tag{36}$$

for the gauge bosons and scalars, respectively. Here s counts the scalars or pseudocscalars with mass M_s depending on $\bar\sigma$ and $\bar\xi$. Additional uncertainties arise from the possible $(\bar\sigma,\bar\xi)$-dependence of the wave function renormalizations $Z_{\sigma,\xi}$ and g. We take here Z_σ and Z_ξ constant and use $\partial g/\partial\bar\xi = \eta_g g, \partial g/\partial\bar\sigma = 0$. As long as $\bar\xi^A,\ A > 0$, acts as an infrared cutoff for the gauge boson fluctuations, the quantity $\eta_g g$ is related to the non-perturbative β-function of g. From asymptotic freedom one expects η_g to be negative. Typically $|\eta_g(g)|$ increases from small perturbative values $\sim g^2$ for small g to large values in the range of large g. The effective $\bar\xi$-dependent gluon mass obeys $\partial M_\rho(\bar\xi)/\partial\bar\xi = (1+\eta_g)M_\rho(\bar\xi)$ and therefore has a minimum for $\eta_g = -1$. One concludes that $\Delta_g U$ is probably minimal for $\bar\xi > 0$, namely for $\eta_g(g(\bar\xi)) = -1$. For $\eta_g(\bar\xi \to 0) < -1$ also the gauge boson fluctuations favor color symmetry breaking. For our numerical solution we show the results for $\eta_g(\langle\bar\xi\rangle) = -1.9, -1.15$. (We mention that for $\eta_g(\langle\bar\xi\rangle) = -1$ the contribution from $\Delta_g U$ drops out in the solution of the field equations.)

For the scalar fluctuations we have only included the light pseudoscalar octet. According to (31) and (29) one finds $M_s^2 \sim \bar\sigma/(\bar\sigma^2 + \frac{7}{36}\frac{Z_\xi}{Z_\sigma}\bar\xi^2)$. For given $\bar\sigma$ and Z_ξ/Z_σ the pseudo-Goldstone boson fluctuations $\Delta_s U$ favor again nonzero $\langle\bar\xi\rangle$ since $M_s(\bar\xi\to\infty)\to 0$. Two parameter sets for a numerical solution including the effects of the gauge bosons and pseudo-Goldstone bosons are given in brackets in (22), one with positive and one with negative ζ. (For the last set we use $k = 820$ MeV.) We also have shown above in brackets the corresponding values for various quantities – the absence of brackets indicating that parameters are tuned in order to obtain physical values.

The choice of the parameters in $\tilde U_k$ for which we have presented results is somewhat arbitrary – we also have found acceptable solutions for rather different sets. The phenomenologically acceptable couplings seem not unnatural and we conclude that spontaneous color symmetry breaking is a plausible phenomenon in QCD. Two features seem characteristic for multiquark interactions that lead to phenomenologically acceptable solutions with the observed values of $M_8, M_1, M_\pi, M_K, M_\eta, M_{\eta'}$ and M_ρ. The first is a large ratio $\lambda_\sigma/\lambda_\chi$. This may be explained by the different running of the effective couplings λ_σ and λ_χ. The second is the large value of the effective gauge coupling g between the gluons and color octet bilinear $\xi\sim\tilde\chi$. At first sight this seems somewhat surprising in view of the fact that $M_\rho \approx 850$ MeV is expected to act as an infrared cutoff. Extrapolations of the two- or three-loop β-functions in the $\overline{MS}$-scheme into this domain would yield substantially smaller values of $\alpha_s(M_\rho)$. An understanding of this issue needs, however, information about the relation of our effective coupling g to the perturbative gauge coupling in the $\overline{MS}$ scheme, and about the running of g in the non-perturbative domain. A first discussion of this issue can be found in [2]. We find it reassuring that the gauge boson fluctuations seem not to play a dominant role for the dynamics of spontaneous color symmetry breaking.

In conclusion, we have presented a consistent mean field picture for gluon-meson duality. The quantitative details are not yet all settled. They would need a computation of the multiquark interactions $\tilde{U}_k$. Without such a computation in the framework of QCD no definite conclusion on the issue of "spontaneous color symmetry breaking" can be drawn. Nevertheless, it is remarkable that for reasonable multiquark couplings a simple scenario can successfully describe all masses of the light pseudoscalar and vector mesons and the baryons, as well as their interactions.

We have identified three mechanisms by which fluctuations operate in favor of a nonvanishing expectation value $\langle\bar{\xi}\rangle$ for the color octet quark-antiquark condensate (in a fixed gauge). Perhaps the most important one arises from the fermion fluctuations which favor large baryon masses and therefore nonzero $\bar{\sigma}, \bar{\xi}$ due to the minus sign in (19). They induce a negative contribution to the quadratic term $\sim \bar{\xi}^2$ in the effective potential. The second ingredient is due to the fluctuations of the (pseudo-)Goldstone bosons. Now $\Delta_s U$ favors small meson masses or a large decay constant f, and therefore again large $\bar{\sigma}, \bar{\xi}$. Finally, a third mechanism is possible if the gluon mass is minimal for a nonzero value of $\langle\bar{\xi}\rangle$. This last issue depends on unknown properties of the non-perturbative running of the gauge coupling and is therefore less solid than the first two mechanisms. The combined effect of these mechanisms has to compete with the "classical mass terms" $\sim \lambda_\sigma, \lambda_\chi$ which favor the absence of spontaneous chiral and color symmetry breaking. The crucial dynamical question is whether the fluctuation effects are strong enough. Finally, we also mention a fourth mechanism which becomes possible for $\zeta < 0$. An increase in $\langle\bar{\sigma}\rangle$ due to fluctuations lowers the mass term for $\bar{\xi}$ if $\zeta < -9\tau_\gamma\bar{\sigma}$ (17). We find it plausible that spontaneous color symmetry breaking indeed occurs in QCD.

The quantitative validity of mean field theory remains questionable in the strongly non-perturbative domain discussed in this note. Mean field theory should, however, indicate at least the qualitative tendencies of the fluctuation effects. We have included here the most important fluctuations of the light baryons and mesons. Furthermore, we find that the term $2\lambda_\sigma\bar{\sigma}$ accounts for 82(99,95) precent of the octet mass M_8. This suggests that the neglected effects of the residual multi-quark interactions after bosonization are subleading. Finally, the existence of a local minimum with $\langle\bar{\sigma}\rangle \neq 0$, $\langle\bar{\xi}\rangle \neq 0$, as found in this note, only requires the validity of mean field theory in a neighborhood around this minimum. Statements of this type should be more robust than a mean field computation of global properties of the effective potential for all $\bar{\sigma}, \bar{\xi}$. We recall that the main shortcoming of mean field theory is that it neglects the effective running of the couplings, as fluctuations with different momenta are included. In the vicinity of the minimum at $(\langle\bar{\sigma}\rangle, \langle\bar{\xi}\rangle)$ the only excitations with mass much smaller than k are the pions.

It would be interesting if some qualitative results of this investigation, namely that the color octet condensate $\langle\bar{\xi}\rangle$ dominates M_ρ, f and $M_1 - M_8$, whereas the singlet condensate $\langle\bar{\sigma}\rangle$ essentially determines M_8 and the combination $M_s^2 f^2$ for the pseudoscalars, could be seen in future lattice simulations. This may be possible indirectly by a study of the quark mass dependence of the meson and baryon masses.

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